Algebraic expressions often contain exponential notation. In this section, we learn how to use exponential notation as well as rules for the order of operations in performing certain algebraic manipulations.

**Exponential Notation**

A product like $3 \cdot 3 \cdot 3 \cdot 3$, in which the factors are the same, is called a **power**. Powers occur often enough that a simpler notation called **exponential notation** is used. For

$$3 \cdot 3 \cdot 3 \cdot 3,$$

we write $3^4$. Because $3^4 = 81$, we can say that 81 is a power of 3.

This is read “three to the fourth power,” or simply, “three to the fourth.” The number 4 is called an **exponent** and the number 3 a **base**.

Expressions like $s^2$ and $s^3$ are usually read “$s$ squared” and “$s$ cubed,” respectively. This comes from the fact that a square with sides of length $s$ has an area $A$ given by $A = s^2$ and a cube with sides of length $s$ has a volume $V$ given by $V = s^3$.

**EXAMPLE 1**

Write exponential notation for $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$.

**SOLUTION**

Exponential notation is $10^5$. 5 is the exponent. 10 is the base.

**EXAMPLE 2**

Simplify: (a) $5^2$; (b) $(−5)^3$; (c) $(2n)^3$.

**SOLUTION**

a) $5^2 = 5 \cdot 5 = 25$ The exponent 2 indicates two factors of 5.

b) $(−5)^3 = (−5)(−5)(−5)$ The exponent 3 indicates three factors of $−5$.

Using the associative law of multiplication

$$= 25(−5)$$

$$= −125$$

c) $(2n)^3 = (2n)(2n)(2n)$ The exponent 3 indicates three factors of $2n$.

Using the associative and commutative laws of multiplication

$$= 2 \cdot 2 \cdot 2 \cdot n \cdot n \cdot n$$

$$= 8n^3$$
To determine what the exponent 1 will mean, look for a pattern in the following:

\[
\begin{align*}
7 \cdot 7 \cdot 7 &= 7^3 \\
7 \cdot 7 &= 7^2 \\
7 &= 7^1 \\
? &= 7^1
\end{align*}
\]

The exponent decreases by 1 each time.

The number of factors decreases by 1 each time. To extend the pattern, we say that

\[7 = 7^1.\]

### Order of Operations

How should \(4 + 2 \times 5\) be computed? If we multiply 2 by 5 and then add 4, the result is 14. If we add 2 and 4 first and then multiply by 5, the result is 30. Since these results differ, the order in which we perform operations matters. If grouping symbols such as parentheses ( ), brackets [ ], braces { }, absolute-value symbols | |, or fraction bars appear, they tell us what to do first. For example,

\[(4 + 2) \times 5\]

indicates \(6 \times 5\), resulting in 30, and

\[4 + (2 \times 5)\]

indicates \(4 + 10\), resulting in 14.

Besides grouping symbols, the following conventions exist for determining the order in which operations should be performed.

### Rules for Order of Operations

1. Calculate within the innermost grouping symbols, ( ), [ ], { }, | |, and above or below fraction bars.
2. Simplify all exponential expressions.
3. Perform all multiplications and divisions, working from left to right.
4. Perform all additions and subtractions, working from left to right.

Thus the correct way to compute \(4 + 2 \times 5\) is to first multiply 2 by 5 and then add 4. The result is 14.

### Example 3

Simplify: \(15 - 2 \cdot 5 + 3\).

**Solution** When no groupings or exponents appear, we always multiply or divide before adding or subtracting:

\[
15 - 2 \cdot 5 + 3 = 15 - 10 + 3 = 5 + 3 = 8.
\]
Always calculate within parentheses first. When there are exponents and no parentheses, we simplify powers before multiplying or dividing.

**EXAMPLE 4**

Simplify: (a) \((3 \cdot 4)^2\); (b) \(3 \cdot 4^2\).

**SOLUTION**

a) \((3 \cdot 4)^2 = (12)^2\) Working within parentheses first
   
   \[= 144\]

b) \(3 \cdot 4^2 = 3 \cdot 16\) Simplifying the power
   
   \[= 48\] Multiplying

Note that \((3 \cdot 4)^2 \neq 3 \cdot 4^2\).

**CAUTION!** Example 4 illustrates that, in general, \((ab)^2 \neq ab^2\).

**EXAMPLE 5**

Evaluate when \(x = 5\): (a) \((-x)^2\); (b) \(-x^2\).

**SOLUTION**

a) \((-x)^2 = (-5)^2 = (-5)(-5) = 25\) We square the opposite of 5.

b) \(-x^2 = -5^2 = -25\) We square 5 and then find the opposite.

**CAUTION!** Example 5 illustrates that, in general, \((-x)^2 \neq -x^2\).

To simplify \(-x^2\), it may help to write

\[-x^2 = (-1)x^2\]

Evaluate \(-15 \div 3(6 - a)^3\) when \(a = 4\).

**SOLUTION**

\[
-15 \div 3(6 - a)^3 = -15 \div 3(6 - 4)^3
\]

Substituting 4 for \(a\)

\[
= -15 \div 3(2)^3
\]

Working within parentheses first

\[
= -15 \div 3 \cdot 8
\]

Simplifying the exponential expression

\[
= -5 \cdot 8
\]

Dividing and multiplying from left to right

\[
= -40
\]

The symbols ( ), [ ], and { } are all used in the same way. Used inside or next to each other, they make it easier to locate the left and right sides of a grouping. When combinations of grouping symbols are used, we begin with the innermost grouping symbols and work to the outside.
EXAMPLE 7
Simplify: \(8 + 4 + 3[9 + 2(3 - 5)^3]\).

**SOLUTION**

\[
8 + 4 + 3[9 + 2(3 - 5)^3] = 8 + 4 + 3[9 + 2(-2)^3]
\]

Doing the calculations in the innermost grouping symbols first

\[
(-2)^3 = (-2)(-2)(-2) = -8
\]

\[
= 8 + 4 + 3[9 + (-16)]
\]

\[
= 8 + 4 + 3[-7]
\]

Completing the calculations within the brackets

\[
= 2 + (-21)
\]

Multiplying and dividing from left to right

\[
= -19
\]

EXAMPLE 8
Calculate: \(\frac{12(9 - 7) + 4 \cdot 5}{3^4 + 2^3}\).

**SOLUTION**

An equivalent expression with brackets is

\[
\left[\frac{12(9 - 7) + 4 \cdot 5}{3^4 + 2^3}\right].
\]

Here the grouping symbols are necessary.

In effect, we need to simplify the numerator, simplify the denominator, and then divide the results:

\[
\frac{12(9 - 7) + 4 \cdot 5}{3^4 + 2^3} = \frac{12(2) + 4 \cdot 5}{81 + 8}
\]

\[
= \frac{24 + 20}{89} = \frac{44}{89}.
\]

Simplifying and the Distributive Law

Sometimes we cannot simplify within grouping symbols. When a sum or a difference is being grouped, the distributive law provides a method for removing the grouping symbols.

EXAMPLE 9
Simplify: \(5x - 9 + 2(4x + 5)\).

**SOLUTION**

\[
5x - 9 + 2(4x + 5) = 5x - 9 + 8x + 10
\]

Using the distributive law

\[
= 13x + 1
\]

Combining like terms

Now that exponents have been introduced, we can make our definition of like or similar terms more precise. Like, or similar, terms are either constant terms or terms containing the same variable(s) raised to the same power(s). Thus, 5 and -7, 19xy and 2yx, and \(4a^3b\) and \(a^3b\) are all pairs of like terms.
EXAMPLE 10  
Simplify: $7x^2 + 3[x^2 + 2x] - 5x$.

SOLUTION

$7x^2 + 3[x^2 + 2x] - 5x = 7x^2 + 3x^2 + 6x - 5x$  
Using the distributive law

$= 10x^2 + x$  
Combining like terms

The Opposite of a Sum

When a number is multiplied by $-1$, the result is the opposite of that number. For example, $-1(7) = -7$ and $-1(-5) = 5$.

The Property of $-1$

For any real number $a$,

$-1 \cdot a = -a$.

(Negative one times $a$ is the opposite of $a$.)

An expression such as $-(x + y)$ indicates the opposite, or additive inverse, of the sum of $x$ and $y$. When a sum within grouping symbols is preceded by a "$-$" symbol, we can multiply the sum by $-1$ and use the distributive law. In this manner, we can find an equivalent expression for the opposite of a sum.

EXAMPLE 11  
Write an expression equivalent to $-(3x + 2y + 4)$ without using parentheses.

SOLUTION

$-(3x + 2y + 4) = -1(3x + 2y + 4)$  
Using the property of $-1$

$= -1(3x) + (-1)(2y) + (-1)(4)$  
Using the distributive law

$= -3x - 2y - 4$  
Using the associative law and the property of $-1$

Example 11 illustrates an important property of real numbers.

The Opposite of a Sum

For any real numbers $a$ and $b$,

$-(a + b) = -a + (-b) = -a - b$.

(The opposite of a sum is the sum of the opposites.)

To remove parentheses from an expression like $-(x - 7y + 5)$, we can first rewrite the subtraction as addition:

$-(x - 7y + 5) = -(x + (-7y) + 5)$  
Rewriting as addition

$= -x + 7y - 5$.  
Taking the opposite of a sum
This procedure is normally streamlined to one step in which we find the opposite by “removing parentheses and changing the sign of every term”:

\[-(x - 7y + 5) = -x + 7y - 5.\]

**EXAMPLE 12**

Simplify: \(3x - (4x + 2)\).

**SOLUTION**

\[
3x - (4x + 2) = 3x + [- (4x + 2)] \quad \text{Adding the opposite of } 4x + 2
\]
\[
= 3x + [-4x - 2] \quad \text{Taking the opposite of } 4x + 2
\]
\[
= 3x + (-4x) + (-2) \quad \text{Try to go directly to this step.}
\]
\[
= 3x - 4x - 2
\]
\[
= -x - 2 \quad \text{Combining like terms}
\]

In practice, the first three steps of Example 12 are generally skipped.

**EXAMPLE 13**

Simplify: \(5t^2 - 2t - (-4t^2 + 9t)\).

**SOLUTION**

\[
5t^2 - 2t - (-4t^2 + 9t) = 5t^2 - 2t + 4t^2 - 9t \quad \text{Removing parentheses and changing the sign of each term inside}
\]
\[
= 9t^2 - 11t \quad \text{Combining like terms}
\]

Expressions such as \(7 - 3(x + 2)\) can be simplified as follows:

\[
7 - 3(x + 2) = 7 + [-3(x + 2)] \quad \text{Adding the opposite of } 3(x + 2)
\]
\[
= 7 + [-3x - 6] \quad \text{Multiplying } x + 2 \text{ by } -3
\]
\[
= 7 - 3x - 6 \quad \text{Try to go directly to this step.}
\]
\[
= 1 - 3x. \quad \text{Combining like terms}
\]

**EXAMPLE 14**

Simplify: (a) \(3n - 2(4n - 5)\); (b) \(7x^3 + 2 - [5(x^3 - 1) + 8]\).

**SOLUTION**

a) \(3n - 2(4n - 5) = 3n - 8n + 10\) \quad \text{Multiplying each term inside the parentheses by } -2
\[
= -5n + 10 \quad \text{Combining like terms}
\]

b) \(7x^3 + 2 - [5(x^3 - 1) + 8] = 7x^3 + 2 - [5x^3 - 5 + 8]\) \quad \text{Removing parentheses}
\[
= 7x^3 + 2 - [5x^3 + 3] \quad \text{Removing brackets}
\]
\[
= 7x^3 + 2 - 5x^3 - 3 \quad \text{Combining like terms}
\]
\[
= 2x^3 - 1
\]

As we progress through our study of algebra, it is important that we be able to distinguish between the two tasks of simplifying an expression and solving an equation. In Chapter 1, we have not solved equations, but we have simplified expressions. This enabled us to write equivalent expressions that were simpler than the given expression. In Chapter 2, we will continue to simplify expressions, but we will also begin to solve equations.