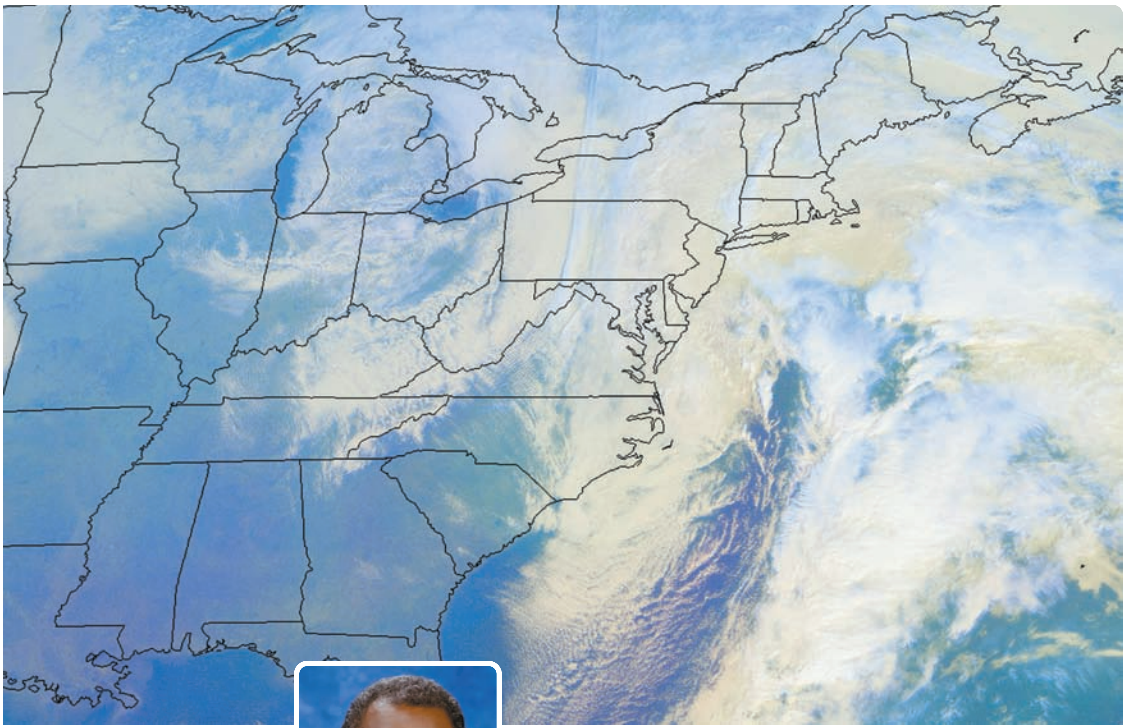
Introduction to Algebraic Expressions



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All weather measurements are a series of numbers and values. Temperature, relative humidity, wind speed and direction, precipitation amount, and air pressure are all expressed in various numbers and percentages. Because weather systems move north and south, east and west, up and down, *and* over time, high-level math like calculus is the only way to represent that movement. But before you study calculus, you must begin with algebra.

AN APPLICATION

On December 10, Jenna notes that the temperature is $-3°$F at 6:00 A.M. She predicts that the temperature will rise at a rate of 2° per hour for 3 hr, and then rise at a rate of 3° per hour for 6 hr. She also predicts that the temperature will then fall at a rate of 2° per hour for 3 hr, and then fall at a rate of 5° per hour for 2 hr. What is Jenna's temperature forecast for 8:00 P.M.?

This problem appears as Exercise 135 in Section 1.7.

CHAPTER 1

Problem solving is the focus of this text. Chapter 1 presents important preliminaries that are needed for the problem-solving approach that is developed in Chapter 2 and used throughout the rest of the book. These preliminaries include a review of arithmetic, a discussion of real numbers and their properties, and an examination of how real numbers are added, subtracted, multiplied, divided, and raised to powers.

1.1 Introduction to Algebra

Algebraic Expressions ▪ Translating to Algebraic Expressions ▪ Translating to Equations

This section introduces some basic concepts and expressions used in algebra. Solving real-world problems is an important part of algebra, so we will focus on the wordings and mathematical expressions that often arise in applications.

Algebraic Expressions

Probably the greatest difference between arithmetic and algebra is the use of *variables* in algebra. When a letter can be any one of a set of numbers, that letter is a **variable**. For example, if n represents the number of tickets purchased for a Maroon 5 concert, then n will vary, depending on factors like price and day of the week. This makes n a variable. If each ticket costs \$40, then 3 tickets cost $40 \cdot 3$ dollars, 4 tickets cost $40 \cdot 4$ dollars, and n tickets cost $40 \cdot n$, or $40n$ dollars. Note that both $40 \cdot n$ and $40n$ mean 40 *times* n. The number 40 is an example of a **constant** because it does not change.

Price per Ticket (in dollars)	Number of Tickets Purchased	Total Paid (in dollars)
40	n	$40n$

The expression $40n$ is a **variable expression** because its value varies with the replacement for n. In this case, the total amount paid, $40n$, will change with the number of tickets purchased. In the following chart, we replace n with a variety of values and compute the total amount paid. In doing so, we are **evaluating the expression** $40n$.

Price per Ticket (in dollars), 40	Number of Tickets Purchased, *n*	Total Paid (in dollars), 40*n*
40	400	$16,000
40	500	20,000
40	600	24,000

Variable expressions are examples of *algebraic expressions*. An **algebraic expression** consists of variables and/or numerals, often with operation signs and grouping symbols. Examples are

$$t + 97, \quad 5 \cdot x, \quad 3a - b, \quad 18 \div y, \quad \frac{9}{7}, \quad \text{and} \quad 4r(s + t).$$

Recall that a fraction bar is a division symbol: $\frac{9}{7}$, or $9/7$, means $9 \div 7$. Similarly, multiplication can be written in several ways. For example, "5 times x" can be written as $5 \cdot x$, $5 \times x$, $5(x)$, or simply $5x$. On many calculators, this appears as $5 * x$.

To **evaluate** an algebraic expression, we substitute a number for each variable in the expression. We then calculate the result.

EXAMPLE 1

STUDENT NOTES

As we will see later, it is sometimes necessary to use parentheses when substituting a number for a variable. You may wish to use parentheses whenever you substitute. In Example 1, we could write

$$x + y = (37) + (28) = 65$$

and

$$5ab = 5(2)(3) = 30.$$

Evaluate each expression for the given values.

a) $x + y$ for $x = 37$ and $y = 28$

b) $5ab$ for $a = 2$ and $b = 3$

SOLUTION

a) We substitute 37 for x and 28 for y and carry out the addition:

$$x + y = 37 + 28 = 65.$$

The number 65 is called the **value** of the expression.

b) We substitute 2 for a and 3 for b and multiply:

$$5ab = 5 \cdot 2 \cdot 3 = 10 \cdot 3 = 30. \qquad 5ab \text{ means 5 times } a \text{ times } b.$$

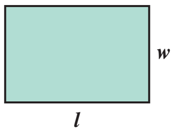
TRY EXERCISE 17

EXAMPLE 2

The area A of a rectangle of length l and width w is given by the formula $A = lw$. Find the area when l is 17 in. and w is 10 in.

SOLUTION We evaluate, using 17 in. for l and 10 in. for w, and carry out the multiplication:

$$A = lw$$
$$A = (17 \text{ in.})(10 \text{ in.})$$
$$A = (17)(10)(\text{in.})(\text{in.})$$
$$A = 170 \text{ in}^2, \text{ or } 170 \text{ square inches.}$$



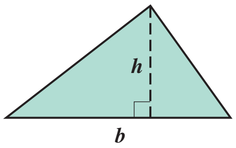
Note that we always use square units for area and $(\text{in.})(\text{in.}) = \text{in}^2$. Exponents like the 2 within the expression in^2 are discussed further in Section 1.8.

TRY EXERCISE 25

EXAMPLE 3 The area of a triangle with a base of length b and a height of length h is given by the formula $A = \frac{1}{2}bh$. Find the area when b is 8 m (meters) and h is 6.4 m.

SOLUTION We substitute 8 m for b and 6.4 m for h and then multiply:

$$A = \tfrac{1}{2}bh$$
$$A = \tfrac{1}{2}(8\text{ m})(6.4\text{ m})$$
$$A = \tfrac{1}{2}(8)(6.4)(\text{m})(\text{m})$$
$$A = 4(6.4)\text{ m}^2$$
$$A = 25.6\text{ m}^2\text{, or 25.6 square meters.}$$



TRY EXERCISE 27

Translating to Algebraic Expressions

Before attempting to translate problems to equations, we need to be able to translate certain phrases to algebraic expressions.

Important Words	Sample Phrase or Sentence	Translation
Addition (+)		
added to	700 pounds was added to the car's weight.	$w + 700$
sum of	The sum of a number and 12	$n + 12$
plus	53 plus some number	$53 + x$
more than	800 more than Biloxi's population	$p + 800$
increased by	Ty's original estimate, increased by 4	$n + 4$
Subtraction (−)		
subtracted from	2 ounces was subtracted from the bag's weight.	$w - 2$
difference of	The difference of two scores	$m - n$
minus	A team of size s, minus 2 injured players	$s - 2$
less than	9 less than the number of volunteers last month	$v - 9$
decreased by	The car's speed, decreased by 8 mph	$s - 8$
Multiplication (·)		
multiplied by	The number of reservations, multiplied by 3	$r \cdot 3$
product of	The product of two numbers	$m \cdot n$
times	5 times the dog's weight	$5w$
twice	Twice the wholesale cost	$2c$
of	$\frac{1}{2}$ of Amelia's salary	$\frac{1}{2}s$
Division (÷)		
divided by	A 2-pound coffee cake, divided by 3	$2 \div 3$
quotient of	The quotient of 14 and 7	$14 \div 7$
divided into	4 divided into the delivery fee	$f \div 4$
ratio of	The ratio of $500 to the price of a new car	$500/p$
per	There were 18 computers per class of size s.	$18/s$

Any variable can be used to represent an unknown quantity; however, it is helpful to choose a descriptive letter. For example, w suggests weight and p suggests population or price. It is important to write down what the chosen variable represents.

EXAMPLE 4 Translate each phrase to an algebraic expression.

a) Four less than Ava's height, in inches

b) Eighteen more than a number

c) A day's pay, in dollars, divided by eight

SOLUTION To help think through a translation, we sometimes begin with a specific number in place of a variable.

a) If the height were 60, then 4 less than 60 would mean $60 - 4$. If the height were 65, the translation would be $65 - 4$. If we use h to represent "Ava's height, in inches," the translation of "Four less than Ava's height, in inches" is $h - 4$.

b) If we knew the number to be 10, the translation would be $10 + 18$, or $18 + 10$. If we use t to represent "a number," the translation of "Eighteen more than a number" is

$$t + 18, \quad \text{or} \quad 18 + t.$$

c) We let d represent "a day's pay, in dollars." If the pay were \$78, the translation would be $78 \div 8$, or $\frac{78}{8}$. Thus our translation of "A day's pay, in dollars, divided by eight" is

$$d \div 8, \quad \text{or} \quad \frac{d}{8}.$$

TRY EXERCISE 31

> ***CAUTION!*** The order in which we subtract and divide affects the answer! Answering $4 - h$ or $8 \div d$ in Examples 4(a) and 4(c) is incorrect.

EXAMPLE 5 Translate each phrase to an algebraic expression.

a) Half of some number

b) Seven more than twice the weight

c) Six less than the product of two numbers

d) Nine times the difference of a number and 10

e) Eighty-two percent of last year's enrollment

SOLUTION

	Phrase	*Variable(s)*	*Algebraic Expression*
a)	Half of some number	Let n represent the number.	$\frac{1}{2}n$, or $\frac{n}{2}$, or $n \div 2$
b)	Seven more than twice the weight	Let w represent the weight.	$2w + 7$, or $7 + 2w$
c)	Six less than the product of two numbers	Let m and n represent the numbers.	$mn - 6$
d)	Nine times the difference of a number and 10	Let a represent the number.	$9(a - 10)$
e)	Eighty-two percent of last year's enrollment	Let r represent last year's enrollment.	82% of r, or $0.82r$

TRY EXERCISE 45

Translating to Equations

The symbol = ("equals") indicates that the expressions on either side of the equals sign represent the same number. An **equation** is a number sentence with the verb =. Equations may be true, false, or neither true nor false.

EXAMPLE 6 Determine whether each equation is true, false, or neither.

a) $8 \cdot 4 = 32$ **b)** $7 - 2 = 4$ **c)** $x + 6 = 13$

SOLUTION

a) $8 \cdot 4 = 32$ The equation is *true*.

b) $7 - 2 = 4$ The equation is *false*.

c) $x + 6 = 13$ The equation is *neither* true nor false, because we do not know what number x represents.

Solution

A replacement or substitution that makes an equation true is called a *solution*. Some equations have more than one solution, and some have no solution. When all solutions have been found, we have *solved* the equation.

To see if a number is a solution, we evaluate all expressions in the equation. If the values on both sides of the equation are the same, the number is a solution.

EXAMPLE 7 Determine whether 7 is a solution of $x + 6 = 13$.

SOLUTION We evaluate $x + 6$ and compare both sides of the equation.

$x + 6 = 13$		Writing the equation
$7 + 6$	13	Substituting 7 for x
$13 \stackrel{?}{=} 13$		13 = 13 is TRUE.

Since the left-hand side and the right-hand side are the same, 7 is a solution.

TRY EXERCISE 57

Although we do not study solving equations until Chapter 2, we can translate certain problem situations to equations now. The words "is the same as," "equal," "is," and "are" often translate to "=."

Words indicating equality, = : "is the same as," "equal," "is," "are"

When translating a problem to an equation, we translate phrases to algebraic expressions, and the entire statement to an equation containing those expressions.

EXAMPLE 8 Translate the following problem to an equation.

What number plus 478 is 1019?

SOLUTION We let y represent the unknown number. The translation then comes almost directly from the English sentence.

What number	plus	478	is	1019?
↓	↓	↓	↓	↓
y	$+$	478	$=$	1019

Note that "what number plus 478" translates to "$y + 478$" and "is" translates to "=."

TRY EXERCISE 63

Sometimes it helps to reword a problem before translating.

EXAMPLE 9 Translate the following problem to an equation.

The Taipei Financial Center, or Taipei 101, in Taiwan is the world's tallest building. At 1666 ft, it is 183 ft taller than the Petronas Twin Towers in Kuala Lumpur. How tall are the Petronas Twin Towers?

Source: *Guinness World Records* 2007

SOLUTION We let h represent the height, in feet, of the Petronas Towers. A rewording and translation follow:

Rewording:	The height of Taipei 101	is	183 ft more than the height of the Petronas Towers
	↓	↓	↓
Translating:	1666	$=$	$h + 183$

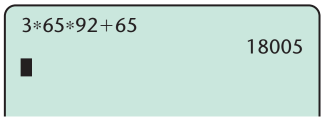
TRY EXERCISE 69

Technology Connections are activities that make use of features that are common to most graphing calculators. In some cases, students may find the user's manual for their particular calculator helpful for exact keystrokes.

Although all graphing calculators are not the same, most share the following characteristics.

Screen. The large screen can show graphs and tables as well as the expressions entered. The screen has a different layout for different functions. Computations are performed in the **home screen**. On many calculators, the home screen is accessed by pressing [2ND] [QUIT]. The **cursor** shows location on the screen, and the **contrast** (set by [2ND] [▲] or [2ND] [▼]) determines how dark the characters appear.

Keypad. There are options written above the keys as well as on them. To access those above the keys, we press [2ND] or [ALPHA] and then the key. Expressions are usually entered as they would appear in print. For example, to evaluate $3xy + x$ for $x = 65$ and $y = 92$, we press 3 [×] 65 [×] 92 [+] 65 and then [ENTER]. The value of the expression, 18005, will appear at the right of the screen.



Evaluate each of the following.

1. $27a - 18b$, for $a = 136$ and $b = 13$
2. $19xy - 9x + 13y$, for $x = 87$ and $y = 29$

STUDY SKILLS

Get the Facts

Throughout this textbook, you will find a feature called Study Skills. These tips are intended to help improve your math study skills. On the first day of class, you should complete this chart.

Instructor: Name ______________________

Office hours and location ______________________

Phone number ______________________

Fax number ______________________

E-mail address ______________________

Find the names of two students whom you could contact for information or study questions:

1. Name ______________________

Phone number ______________________

E-mail address ______________________

2. Name ______________________

Phone number ______________________

E-mail address ______________________

Math lab on campus:

Location ______________________

Hours ______________________

Phone ______________________

Tutoring:

Campus location ______________________

Hours ______________________

Important supplements:

(See the preface for a complete list of available supplements.)

Supplements recommended by the instructor.

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Translating for Success

Translate to an expression or an equation and match that translation with one of the choices A–O below. Do not solve.

1. Twice the difference of a number and 11

2. The product of a number and 11 is 2.

3. Twice the difference of two numbers is 11.

4. The quotient of twice a number and 11

5. The quotient of 11 and the product of two numbers

6. Eleven times the sum of a number and twice the number

7. Twice the sum of two numbers is 11.

8. Two more than twice a number is 11.

9. Twice the sum of 11 times a number and 2

10. Twenty percent of some number is 11.

A. $x = 0.2(11)$

B. $\frac{2x}{11}$

C. $2x + 2 = 11$

D. $2(11x + 2)$

E. $11x = 2$

F. $0.2x = 11$

G. $11(2x - y)$

H. $2(x - 11)$

I. $11 + 2x = 2$

J. $2x + y = 11$

K. $2(x - y) = 11$

L. $11(x + 2x)$

M. $2(x + y) = 11$

N. $2 + \frac{x}{11}$

O. $\frac{11}{xy}$

Answers on page A-1

An additional, animated version of this activity appears in MyMathLab. To use MyMathLab, you need a course ID and a student access code. Contact your instructor for more information.