OBJECTIVES
1 Plot Points in the Rectangular Coordinate System
2 Determine Whether an Ordered Pair Is a Point on the Graph of an Equation
3 Graph an Equation Using the Point-Plotting Method
4 Identify the Intercepts from the Graph of an Equation
5 Interpret Graphs

Preparing for Rectangular Coordinates and Graphs of Equations
Before getting started, take this readiness quiz. If you get a problem wrong, go back to the section cited and review the material.

P1. Plot the following points on the real number line: 
-2, 4, 0, \(\frac{1}{2}\).  
[Section R.2, pp. 15–16]

P2. Determine which of the following are solutions to the equation \(3x - 5(x + 2) = 4\).
(a) \(x = 0\)  (b) \(x = -3\)  (c) \(x = -7\)  
[Section 1.1, pp. 48–49]

P3. Evaluate the expression \(2x^2 - 3x + 1\) for the given values of the variable.
(a) \(x = 0\)  (b) \(x = 2\)  (c) \(x = -3\)  
[Section R.5, pp. 40–41]

P4. Solve the equation \(3x + 2y = 8\) for \(y\).  
[Section 1.3, pp. 73–76]

P5. Evaluate \(|-4|\).  
[Section R.3, pp. 19–20]

Plot Points in the Rectangular Coordinate System

Recall from Section R.2 that we locate a point on the real number line by assigning it a single real number, called the coordinate of the point. See Preparing for Problem P1. When we graph a point on the real number line, we are working in one dimension. When we wish to work in two dimensions, we locate a point using two real numbers.

We begin by drawing two real number lines that intersect at right (90°) angles. One of the real number lines is drawn horizontal, while the other is drawn vertical. We call the horizontal real number line the \(x\)-axis, and the vertical real number line is the \(y\)-axis. The point where the \(x\)-axis and \(y\)-axis intersect is called the origin, \(O\). See Figure 23.

![Figure 23](image)

The origin \(O\) has a value of 0 on the \(x\)-axis and the \(y\)-axis. Points on the \(x\)-axis to the right of \(O\) are positive real numbers; points on the \(x\)-axis to the left of \(O\) are negative real numbers. Points on the \(y\)-axis that are above \(O\) are positive real numbers; points on the \(y\)-axis that are below \(O\) are negative real numbers. In Figure 23 we label the \(x\)-axis “\(x\)” and the \(y\)-axis “\(y\)” Notice that an arrow is used at the end of each axis to denote the positive direction. We do not use an arrow to denote the negative direction.

The coordinate system presented in Figure 23 is called a rectangular or Cartesian coordinate system, named after René Descartes (1596–1650), a French mathematician, philosopher, and theologian. The plane formed by the \(x\)-axis and \(y\)-axis is often referred to as the \(xy\)-plane, and the \(x\)-axis and \(y\)-axis are called the coordinate axes.

We can represent any point \(P\) in the rectangular coordinate system by using an ordered pair \((x, y)\) of real numbers. If \(x > 0\), we travel \(x\) units to the right of the \(y\)-axis;
if $x < 0$, we travel $|x|$ units to the left of the $y$-axis. If $y > 0$, we travel $y$ units above the $x$-axis; if $y < 0$, we travel $|y|$ units below the $x$-axis. The ordered pair $(x, y)$ is also called the coordinates of $P$.

The origin $O$ has coordinates $(0, 0)$. Any point on the $x$-axis has coordinates of the form $(x, 0)$, and any point on the $y$-axis has coordinates of the form $(0, y)$.

If $(x, y)$ are the coordinates of a point $P$, then $x$ is called the $x$-coordinate, or abscissa, of $P$ and $y$ is called the $y$-coordinate or ordinate, of $P$.

If you look back at Figure 23, you should notice that the $x$- and $y$-axes divide the plane into four separate regions or quadrants. In quadrant I, both the $x$- and $y$-coordinate are positive; in quadrant II, $x$ is negative and $y$ is positive; in quadrant III, both $x$ and $y$ are negative; and in quadrant IV, $x$ is positive and $y$ is negative. Points on the coordinate axes do not belong to a quadrant. See Figure 24.

**EXAMPLE 1** Plotting Points in the Rectangular Coordinate System and Determining the Quadrant in which the Point Lies

Plot the points in the $xy$-plane. Tell which quadrant each point is in.

(a) $A(3, 2)$  
(b) $B(-2, 4)$  
(c) $C(-1, -3)$  
(d) $D(3, -4)$  
(e) $E(-2, 0)$

**Solution**

Before we plot the points, we draw a rectangular or Cartesian coordinate system. See Figure 25(a). We now plot the points.

(a) To plot point $A(3, 2)$, from the origin $O$, we travel 3 units to the right and then 2 units up. Label the point $A$. See Figure 25(b). Point $A$ is in quadrant I because both $x$ and $y$ are positive.
(b) To plot point $B(-2, 4)$, from the origin $O$, we travel 2 units to the left and then 4 units up. Label the point $B$. See Figure 25(b). Point $B$ is in quadrant II.

c) See Figure 25(b). Point $C$ is in quadrant III.

d) See Figure 25(b). Point $D$ is in quadrant IV.

e) See Figure 25(b). Point $E$ is not in a quadrant because it lies on the $x$-axis.

Quick

1. The point where the $x$-axis and $y$-axis intersect in the Cartesian coordinate system is called the ________.

2. True or False: If a point lies in quadrant III of the Cartesian coordinate system, then both $x$ and $y$ are negative.

In Problems 3 and 4, plot each point in the $xy$-plane. Tell in which quadrant or on which coordinate axis each point lies.

3. (a) $A(5, 2)$
   (b) $B(4, -2)$
   (c) $C(0, -3)$
   (d) $D(-4, -3)$

4. (a) $A(-3, 2)$
   (b) $B(-4, 0)$
   (c) $C(3, -2)$
   (d) $D(6, 1)$

2 Determine Whether an Ordered Pair Is a Point on the Graph of an Equation

In Section 1.1, we solved linear equations in one variable. The solution was either a single value of the variable, the empty set, or all real numbers. We will now look at equations in two variables. Our goal is to learn a method for representing the solution to an equation in two variables.

DEFINITION

An equation in two variables, say $x$ and $y$, is a statement in which the algebraic expressions involving $x$ and $y$ are equal. The expressions are called sides of the equation.

Since an equation is a statement, it may be true or false, depending upon the values of the variables. Any values of the variable that make the equation a true statement are said to satisfy the equation.

For example, the following are all equations in two variables.

\[ x^2 = y + 2 \quad 3x + 2y = 6 \quad y = -4x + 5 \]

The first equation $x^2 = y + 2$ is satisfied when $x = 3$ and $y = 7$ since $3^2 = 7 + 2$. It is also satisfied when $x = -2$ and $y = 2$. In fact, there are infinitely many choices of $x$ and $y$ that satisfy the equation $x^2 = y + 2$. However, there are some choices of $x$ and $y$ that do not satisfy the equation $x^2 = y + 2$. For example, $x = 3$ and $y = 4$ does not satisfy the equation because $3^2 \neq 4 + 2$ (that is, $9 \neq 6$).

When we find a value of $x$ and $y$ that satisfies an equation, it means that the ordered pair $(x, y)$ represents a point on the graph of the equation.

DEFINITION

The graph of an equation in two variables $x$ and $y$ is the set of all points whose coordinates, $(x, y)$, in the $xy$-plane satisfy the equation.
**EXAMPLE 2** Determining Whether a Point Is on the Graph of an Equation

Determine if the following coordinates represent points that are on the graph of $3x - y = 6$.

(a) $(2, 0)$   (b) $(1, -2)$   (c) $\left(\frac{1}{2}, \frac{-9}{2}\right)$

**Solution**

(a) For $(2, 0)$, we check to see if $x = 2, y = 0$ satisfies the equation $3x - y = 6$.

$$3x - y = 6$$

Let $x = 2, y = 0$: $3(2) - 0 \neq 6$

$$6 \neq 6 \quad \text{False}$$

The statement is true when $x = 2$ and $y = 0$, so the point whose coordinates are $(2, 0)$ is on the graph.

(b) For $(1, -2)$, we have

$$3x - y = 6$$

Let $x = 1, y = -2$: $3(1) - (-2) \neq 6$

$$5 \neq 6 \quad \text{False}$$

The statement $5 = 6$ is false, so the point whose coordinates are $(1, -2)$ is not on the graph.

(c) For $\left(\frac{1}{2}, \frac{-9}{2}\right)$, we have

$$3x - y = 6$$

Let $x = \frac{1}{2}, y = \frac{-9}{2}$: $3\left(\frac{1}{2}\right) - \left(\frac{-9}{2}\right) = 6$

$$\frac{3}{2} + \frac{9}{2} = 6$$

$$6 = 6 \quad \text{True}$$

The statement is true when $x = \frac{1}{2}$ and $y = \frac{-9}{2}$, so the point whose coordinates are $\left(\frac{1}{2}, \frac{-9}{2}\right)$ is on the graph.

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**Quick ✓**

5. *True or False*: The graph of an equation in two variables $x$ and $y$ is the set of all points whose coordinates, $(x, y)$, in the Cartesian plane satisfy the equation.

6. Determine if the following coordinates represent points that are on the graph of $2x - 4y = 12$.

(a) $(2, -3)$   (b) $(2, -2)$   (c) $\left(\frac{3}{2}, \frac{-9}{4}\right)$

7. Determine if the following coordinates represent points that are on the graph of $y = x^2 + 3$.

(a) $(1, 4)$   (b) $(-2, -1)$   (c) $(-3, 12)$

For the remainder of the course we will say “the point $(x, y)$” rather than “the point whose coordinates are $(x, y)$” for the sake of brevity.
Graph an Equation Using the Point-Plotting Method

One of the most elementary methods for graphing an equation is the point-plotting method. With this method, we choose values for one of the variables and use the equation to determine the corresponding values of the remaining variable. If \( x \) and \( y \) are the variables in the equation, it does not matter whether we choose values of \( x \) and use the equation to find the corresponding \( y \) or choose \( y \) and find \( x \). Convenience will determine which way we go.

**EXAMPLE 3 How to Graph an Equation by Plotting Points**

Graph the equation \( y = -2x + 4 \) by plotting points.

**Step-by-Step Solution**

**Step 1:** We want to find all points \((x, y)\) that satisfy the equation. To determine these points we choose values of \( x \) (do you see why?) and use the equation to determine the corresponding values of \( y \). See Table 9.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = -2x + 4 )</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-3)</td>
<td>(-2(-3) + 4 = 10)</td>
<td>((-3, 10))</td>
</tr>
<tr>
<td>(-2)</td>
<td>(-2(-2) + 4 = 8)</td>
<td>((-2, 8))</td>
</tr>
<tr>
<td>(-1)</td>
<td>(-2(-1) + 4 = 6)</td>
<td>((-1, 6))</td>
</tr>
<tr>
<td>(0)</td>
<td>(-2(0) + 4 = 4)</td>
<td>((0, 4))</td>
</tr>
<tr>
<td>(1)</td>
<td>(-2(1) + 4 = 2)</td>
<td>((1, 2))</td>
</tr>
<tr>
<td>(2)</td>
<td>(-2(2) + 4 = 0)</td>
<td>((2, 0))</td>
</tr>
<tr>
<td>(3)</td>
<td>(-2(3) + 4 = -2)</td>
<td>((3, -2))</td>
</tr>
</tbody>
</table>

**Step 2:** We plot the ordered pairs listed in the third column of Table 9 as shown in Figure 26(a). Now connect the points to obtain the graph of the equation (a line) as shown in Figure 26(b).

The graph of the equation shown in Figure 26(b) does not show all the points that satisfy \( y = -2x + 4 \). For example, in Figure 26(b) the point \((8, -12)\) is part of the graph of \( y = -2x + 4 \), but it is not shown. Since the graph of \( y = -2x + 4 \) can be extended as far as we please, we use arrows on the ends of the graph to indicate that the pattern shown continues. It is important to show enough of the graph so that anyone who is looking at it will “see” the rest of it as an obvious continuation of what is there. This is called a complete graph.
EXAMPLE 4 Graphing an Equation by Plotting Points

Graph the equation $y = x^2$ by plotting points.

**Solution**

Table 10 shows several points on the graph.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = x^2$</th>
<th>$(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-4$</td>
<td>$(-4)^2 = 16$</td>
<td>$(-4, 16)$</td>
</tr>
<tr>
<td>$-3$</td>
<td>$(-3)^2 = 9$</td>
<td>$(-3, 9)$</td>
</tr>
<tr>
<td>$-2$</td>
<td>$(-2)^2 = 4$</td>
<td>$(-2, 4)$</td>
</tr>
<tr>
<td>$-1$</td>
<td>$(-1)^2 = 1$</td>
<td>$(-1, 1)$</td>
</tr>
<tr>
<td>$0$</td>
<td>$(0)^2 = 0$</td>
<td>$(0, 0)$</td>
</tr>
<tr>
<td>$1$</td>
<td>$(1)^2 = 1$</td>
<td>$(1, 1)$</td>
</tr>
<tr>
<td>$2$</td>
<td>$(2)^2 = 4$</td>
<td>$(2, 4)$</td>
</tr>
<tr>
<td>$3$</td>
<td>$(3)^2 = 9$</td>
<td>$(3, 9)$</td>
</tr>
<tr>
<td>$4$</td>
<td>$(4)^2 = 16$</td>
<td>$(4, 16)$</td>
</tr>
</tbody>
</table>

In Figure 27(a), we plot the ordered pairs listed in Table 10. In Figure 27(b), we connect the points in a smooth curve.

**Work Smart**

Notice we use a different scale on the $x$- and $y$-axis in Figure 27.

**Work Smart**

Experience will play a huge role in determining which $x$-values to choose in creating a table of values. For the time being, start by choosing values of $x$ around $x = 0$ as in Table 10.

Two questions that you might be asking yourself right now are “How do I know how many points are sufficient?” and “How do I know which $x$-values (or $y$-values) I should choose in order to obtain points on the graph?” Often, the type of equation we wish to graph indicates the number of points that are necessary. For example, we will learn in the next section that if the equation is of the form $y = mx + b$, then its graph is a line and only two points are required to obtain the graph (as in Example 3). Other times, more points are required. At this stage in your math career, you will need to plot quite a few points to obtain a complete graph. However, as your experience and knowledge grow, you will learn to be more efficient in obtaining complete graphs.

**Quick**

In Problems 8–10, graph each equation using the point-plotting method.

8. $y = 3x + 1$

9. $2x + 3y = 8$

10. $y = x^2 + 3$
EXAMPLE 5  Graphing the Equation $x = y^2$

Graph the equation $x = y^2$ by plotting points.

**Solution**

Because the equation is solved for $x$, we will choose values of $y$ and use the equation to find the corresponding values of $x$. See Table 11. We plot the ordered pairs listed in Table 11 and connect the points in a smooth curve. See Figure 28.

<table>
<thead>
<tr>
<th>$y$</th>
<th>$x = y^2$</th>
<th>$(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-3$</td>
<td>$(-3)^2 = 9$</td>
<td>$(9, -3)$</td>
</tr>
<tr>
<td>$-2$</td>
<td>$(-2)^2 = 4$</td>
<td>$(4, -2)$</td>
</tr>
<tr>
<td>$-1$</td>
<td>$(-1)^2 = 1$</td>
<td>$(1, -1)$</td>
</tr>
<tr>
<td>$0$</td>
<td>$0^2 = 0$</td>
<td>$(0, 0)$</td>
</tr>
<tr>
<td>$1$</td>
<td>$1^2 = 1$</td>
<td>$(1, 1)$</td>
</tr>
<tr>
<td>$2$</td>
<td>$2^2 = 4$</td>
<td>$(4, 2)$</td>
</tr>
<tr>
<td>$3$</td>
<td>$3^2 = 9$</td>
<td>$(9, 3)$</td>
</tr>
</tbody>
</table>

Quick ✓ In Problems 11 and 12, graph each equation using the point-plotting method.

11. $x = y^2 + 2$
12. $x = (y - 1)^2$

4  Identify the Intercepts from the Graph of an Equation

One of the key components that should be displayed in a complete graph is the intercepts of the graph.

**DEFINITION**

The intercepts are the coordinates of the points, if any, where a graph crosses or touches the coordinate axes. The $x$-coordinate of a point at which the graph crosses or touches the $x$-axis is an $x$-intercept, and the $y$-coordinate of a point at which the graph crosses or touches the $y$-axis is a $y$-intercept.

See Figure 29 for an illustration. Notice that an $x$-intercept exists when $y = 0$ and a $y$-intercept exists when $x = 0$.

EXAMPLE 6  Finding Intercepts from a Graph

Find the intercepts of the graph shown in Figure 30. What are the $x$-intercepts? What are the $y$-intercepts?
**Solution**

The intercepts of the graph are the points

\((-3, 0), (0, 2), (1, 0), \text{ and } (3.8, 0)\)

The \(x\)-intercepts are \(-3, 1,\) and \(3.8\). The \(y\)-intercept is \(2\).

In Example 6, you should notice the following: If we do not specify the type of intercept (\(x\)- versus \(y\)-), then we report the intercept as an ordered pair. However, if we specify the type of intercept, then we only need to report the coordinate of the intercept. For \(x\)-intercepts, we report the \(x\)-coordinate of the intercept (since it is understood that the \(y\)-coordinate is 0); for \(y\)-intercepts, we report the \(y\)-coordinate of the intercept (since the \(x\)-coordinate is understood to be 0).

**Quick Check**

13. The points, if any, at which a graph crosses or touches a coordinate axis are called _________.

14. True or False: The graph of an equation must have at least one \(x\)-intercept.

15. Find the intercepts of the graph shown in the figure. What are the \(x\)-intercepts? What are the \(y\)-intercepts?

**Interpret Graphs**

Graphs play an important role in helping us to visualize relationships that exist between two variables or quantities. We have all heard the expression “A picture is worth a thousand words.” A graph is a “picture” that illustrates the relationship between two variables. By visualizing this relationship, we are able to see important information and draw conclusions regarding the relationship between the two variables.

**Example 7 Interpret a Graph**

The graph in Figure 31 shows the profit \(P\) for selling \(x\) gallons of gasoline in an hour at a gas station. The vertical axis represents the profit and the horizontal axis represents the number of gallons of gasoline sold.

**Figure 31**
(a) What is the profit if 150 gallons of gasoline are sold?
(b) How many gallons of gasoline are sold when profit is highest? What is the highest profit?
(c) Identify and interpret the intercepts.

**Solution**

(a) Draw a vertical line up from 150 on the horizontal axis until we reach the point on the graph. Then draw a horizontal line from this point to the vertical axis. The point where the horizontal line intersects the vertical axis is the profit when 150 gallons of gasoline are sold. The profit from selling 150 gallons of gasoline is $200.

(b) The profit is highest when 375 gallons of gasoline are sold. The highest profit is $565.

(c) The intercepts are (0, −200), (100, 0), and (750, 0). For (0, −200): If the price of gasoline is too high, demand for gasoline (in theory) will be 0 gallons. This is the explanation for selling 0 gallons of gasoline. The negative profit is due to the fact that the company has $0 in revenue (since it didn’t sell any gas), but had hourly expenses of $200. For (100, 0): The company sells just enough gas to pay its bills. This can be thought of as the break-even point. For (750, 0): The 750 gallons sold represents the maximum number of gallons that the station can pump and still break even.

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**Quick**

16. The graph shown represents the cost \( C \) (in thousands of dollars) of refining \( x \) gallons of gasoline per hour (in thousands). The vertical axis represents the cost and the horizontal axis represents the number of gallons of gasoline refined.

(a) What is the cost of refining 250 thousand gallons of gasoline per hour?
(b) What is the cost of refining 400 thousand gallons of gasoline per hour?
(c) In the context of the problem, explain the meaning of the graph ending at 700 thousand gallons of gasoline.
(d) Identify and interpret the intercept.